2a) $T_v(x_1) = v^T \cdot x_1$ $\qquad$ $T_v(x_2) = v^T \cdot x_2$.

$T_v(x_1 + x_2) = v^T \cdot (x_1 + x_2)$

$T_v(x_1) + T_v(x_2) = v^T x_1 + v^T x_2 = v^T \cdot (x_1 + x_2)$ cumple ✓.

$\alpha \cdot T_v(x_1) = \alpha x_1 \cdot v^T$

$T_v(\alpha x_1) = v^T \cdot \alpha x_1 = \alpha x_1 v^T$ cumple ✓

es funcional lineal.

2.2 b) $T_A(X) : tr(A^*X)$

$TA(Y) : tr(A^*Y)$

$TA(X+Y) = tr(A^*(X+Y)) \to tr(A^*X + A^*Y) \to tr(A^*X) + tr(A^*Y)$

$TA(X) + TA(Y) = tr(A^*X) + tr(A^*Y)$ ✓ cumple.

$TA(\alpha X) = tr(A^*\alpha X) = \alpha \cdot tr(A^*X)$

$\alpha\, TA(X) = \alpha \cdot tr(A^*X)$ cumple ✓.

2c)

$$T_g(F_1) := \int_{-1}^{1} F_1(x)\,\overline{g(x)}\,dx \quad \forall F \in C(\mathbb{R}, \mathbb{C})$$

$$T_g(F_2) = \int_{-1}^{1} F_2(x)\,\overline{g(x)}\,dx$$

$$T_g(F_1+F_2) = \int_{-1}^{1} [F_1(x)+F_2(x)] \cdot \overline{g(x)}\,dx = \int_{-1}^{1} F_1(x) \cdot \overline{g(x)}\,dx + \int_{-1}^{1} F_2(x)\overline{g(x)}\,dx$$

$$T_g(F_1)+T_g(F_2) = \int_{-1}^{1} F_1(x)\overline{g(x)}\,dx + \int_{-1}^{1} F_2(x)\overline{g(x)}\,dx$$

CUMPLE ✓.

$$T_g(\alpha \cdot F_1) = \int_{-1}^{1} \alpha F_1(x)\,\overline{g(x)}\,dx = \alpha \cdot \int_{-1}^{1} F_1(x)\,\overline{g(x)}\,dx = \alpha \cdot T_g(F_1)$$

CUMPLE ✓.

CUMPLE ✓.

d) $T_A(x) := Ax$

$\rightarrow$ $T_A(x_1) = Ax_1$

$T_A(x_2) = Ax_2$

[$T_A(x_1 + x_2) = A(x_1 + x_2) = Ax_1 + Ax_2 = T_A(x_1) + T_A(x_2)$ ✓ CUMPLE

[$T_A(\alpha x_1) = A.(\alpha x_1) = \alpha . Ax_1 = \alpha . T_A(x_1)$ CUMPLE ✓.

e) $L[y_1] := \frac{d^n y_1}{dx^n} + a_{n-1} \frac{d^{n-1} y_1}{dx^{n-1}} + \dots + a_1 \frac{dy_1}{dx} + a_0 . y_1$

$L[y_2] = \frac{d^n y_2}{dx^n} + a_{n-1} . \frac{d^{n-1} y_2}{dx^{n-1}} + \dots + a_1 . \frac{dy_2}{dx} + a_0 . y_2 .$

$\left[L[y_1+y_2] = \frac{d^n (y_1+y_2)}{dx^n} + a_{n-1} \frac{d^{n-1} (y_1+y_2)}{dx^{n-1}} + \dots + a_1 \frac{d(y_1+y_2)}{dx} + a_0 (y_1+y_2) = \right.$

$= \frac{d^n y_1}{dx^n} + \frac{d^n y_2}{dx^n} + a_{n-1} \frac{d^{n-1} y_1}{dx^{n-1}} + a_{n-1} \frac{d^{n-1} y_2}{dx^{n-1}} + \dots + a_1 \frac{dy_1}{dx} + a_1 \frac{dy_2}{dx} + a_0 y_1 + a_0 y_2 =$

$\left. = L[y_1] + L[y_2] \right]$ Cumple ✓.

$\left[L[\alpha y_1] = \frac{d^n \alpha . y_1}{dx^n} + a_{n-1} \frac{d^{n-1} \alpha y_1}{dx^{n-1}} + \dots + a_1 \frac{d(\alpha y_1)}{dx} + a_0 . \alpha y_1 = \right.$

$\left. = \alpha . \left(\frac{d^n y_1}{dx^n} + a_{n-1} \frac{d^{n-1} y_1}{dx^{n-1}} + \dots + a_1 \frac{dy_1}{dx} + a_0 . y_1 \right) = \alpha . L[y_1] \right]$

Cumple ✓